On the conflict between local realism and classical physics

S. Savasta, O. Di Stefano, R. Girlanda

Dipartimento di Fisica della Materia e Tecnologie Fisiche Avanzate,

Università di Messina Salita Sperone 31, I-98166 Messina, Italy

Abstract

In contrast to the intuitively plausible assumption of local realism, entangled particles, even when isolated, are not allowed to possess definite properties in their own right , as quantitatively expressed by violations of Bells inequalities [1]. Even as entanglement is now a key feature of quantum information and communication technology [2, 3], it remains the most puzzling feature of quantum mechanics [4] and its conceptual foundation is still widely debated. Here we demonstrate that physical systems providing dicotomic outcomes are not able to guarantee both the rotation properties of physical quantities and local realism. This result opens the way to a new formulation of quantum mechanics based on only two elementary physical principles replacing the abstract mathematical axiomes of the present theory. According to this formulation, the coexistence of discrete outcomes with the classical continuous transformation properties of physical quantities under coordinate transformation inevitably implies quantum probabilities. These results, provide a simple physical explanation to the most debated quantum features and put into question the existence of physical quantities displaying continuous outcomes in agreement with approaches that attempt to integrate quantum theory with general relativity [5, 6, 7, 8].

Einstein reduced the abstract mathematical structure of the Lorentz transformations to two simple physical principles expressible in common language. Although the theory of special relativity leads to surprising and in part even counterintuitive consequences, thanks to the existence of these physical principles we do not have a significant debate on the interpretation of the theory of special relativity. The formulation of quantum mechanics, to the contrary, is based on a number of rather abstract, axioms. The motivations for the postulates is not always clear, and they appeared surprising even to the founding fathers of quantum theory. This absence of elementary physical principles together with the counterintuitive consequences of the postulates determined a relentless broad discussion about the interpretation of the theory [9], despite its success. Albert Einstein disliked the loss of determinism in measurement. He held that quantum mechanics must be incomplete, and produced a series of objections to the theory. The most famous of these was the Einstein Podolsky Rosen (EPR) paradox [10]. The EPR paradox was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional (hidden) variables to restore causality and locality. Bell's theorem put forward the conflict between quantum mechanics and hidden variables predictions [11]. Later experimental verification of violations of Bell inequalities [12, 13] disproved local realism and hidden variable theories, showing that nature is intrinsically not deterministic as predicted by quantum theory. Nevertheless John Bell himself was profoundly unsatisfied with the axiomatic formulation of quantum theory [4]. According to him ordinary quantum mechanics is just fine for all practical purposes (abbreviated, rather disparagingly, as "FAPP"). Recently the field of quantum information theory opened up and expanded rapidly. Quantum entanglement began to be seen not only as a puzzle, but also as a resource which can yield new physical effects and techniques [3]. In turn ideas from these fields are beginning to yield new insight into the foundations of quantum physics, suggesting that information should play an essential role in the foundations of any scientific description of Nature [14, 15, 16]. Besides, classical physics appears to play a not dismissable role. Despite quantum theory provides probably the most striking departure from classical ideas of reality, it requires for its formulation classical concepts [17]. Here we explore this deep connection by adopting a conservative approach, trying to introduce into classical physics the smallest modifcations able to describe experimental evidences of light quanta.

We start our analysis pointing out a not new and rather obvious contact point between

classical and quantum physics. The polarization status of a monochromatic light beam can be represented by a unit vector $\mathbf{J}$ on the Poincaré sphere. The outcome of polarization measurements along a direction $\mathbf{n}$ on the Poincaré sphere can be described by $\mathbf{J} \cdot \mathbf{n}$. Let us consider an horizontally-polarized light beam. Of course further measurements of polarization in the H/V (horizontal/vertical) basis will say that all the light will follow the H direction ($\mathbf{J} \equiv (0, 0, 1)$). We consider now measurements of linear polarization along directions H'/V' (along the $\mathbf{x}$ axis of the Poincaré sphere) rotated by 45° with respect to the horizontal axis . According to classical physics, the light beam will be splitted into two beams of equal intensities $I_{H'} = I_{V'}$, giving rise to $\mathbf{J} \cdot \mathbf{x} = (I_{H'} - I_{V'})/(I_{H'} + I_{V'}) = 0$. If the beam is progressively attenuated before passing the polarizing beam splitters, nature tells us that the splitting into two beams cannot be continued indefinitely, we reach a situation where a single light quantum will follow one direction only. If the photon cannot be divided which are the new rules? Which direction will it follow? Actually we will see photons choosing either path H' or V' randomly. If we use the experimental outcomes (e.g. the number of photons $n_{H'}$ and $n_{V'}$ detected by the two photon-counters after repeated $N = n_{H'} + n_{V'}$ events), the mean value of the Stokes parameter $(n_{H'} - n_{V'})/N$ will approach zero when increasing N: $\langle \mathbf{J} \cdot \mathbf{x} \rangle = \lim_{N \to \infty}(n_{H'} - n_{V'})/N = 0$, i.e. it will be zero in the mean. Thus $\langle \mathbf{J} \cdot \mathbf{x} \rangle$ follows classical physics. So in a situation where physicists were forced to abandon the ship of classical physics, this contact point can be viewed as a life raft. *Once we accept the presence of discrete outcomes, the emerging of quantum probabilities can be viewed as the only means that nature has to follow the transformation rules of classical physics.*

Let us now consider a system of two particles with zero total angular momentum. According to classical physics this imply that if we measure projections of total angular momentum along an arbitrary axis $\hat{\mathbf{n}}$, we will find zero, so $\mathbf{J} \cdot \hat{\mathbf{n}}$ and $|\mathbf{J}|$ will be zero. We may assume that total angular momentum is conserved even if the two particles are well separated and have ceased to interact. If we also assume that measurements of angular momentum along a given axis take values only in $\{\pm 1\}$, the only possibility for the existence of two particles with zero total angular momentum is that they display perfect anticorrelation for measurements along arbitrary axes, i.e. if particle 1 provides the outcome $v_1(\mathbf{n}) = \pm 1$, the other will provide opposite values so that $v_1(\mathbf{n}) v_2(\mathbf{n}) = -1$. Just for continuity with the previous example, let us consider light quanta. Each observer has his own detection apparatus PBS $\mathbf{n}$ able to perform polarization measurements along a direction $\mathbf{n}$ on the Poincaré sphere. The CHSH inequality

[18], on which many experimental tests of Einstein *local realism* have been performed, is part of the large set of inequalities known generally as Bell inequalities. It applies to a situation in which observer 1 which receives particle 1 can choose to use either PBS $\mathbf{n}$ or PBS $\mathbf{n}'$. Analogously observer 2 can use either PBS $\mathbf{m}$ or PBS $\mathbf{m}'$. The observables v_1 and v_2 take values only in $\{\pm 1\}$ and may be functions of hidden random variables. The inequality can be expressed as $|\langle B\rangle| \leq 2$, where $B \equiv [v_1(\mathbf{n}) + v_1(\mathbf{n}')]\, v_2(\mathbf{m}) + [v_1(\mathbf{n}) - v_1(\mathbf{n}')]\, v_2(\mathbf{m}') = \pm 2$. It poses a limit to the degree of correlation permitted by a theory assuming dicotomic outcomes and local realism. We now ask if the mean values of these variables satisfy (at least statistically) the laws of classical physics as the Stokes parameter examined before. To this aim we consider a system with zero total angular momentum. We start addressing the situation where the two observers perform measurements along the same axis $\mathbf{n}$. Each run of the experiment will give rise to $v_1(\mathbf{n}) v_2(\mathbf{n}) = -1$. What happens if observer 2 uses a differently oriented apparatus PBS $\mathbf{m}$? According to the transformation properties of vectors under rotations, from $v_1(\mathbf{n}) v_2(\mathbf{n}) = -1$, it results [19] $v_1(\mathbf{n}) v_2(\mathbf{m}) = -\mathbf{n}\cdot\mathbf{m}$. Of course theories with discrete outcomes cannot give this result that is not in $\{\pm 1\}$. Driven by the previous example we may require that this result holds in the mean: $\langle v_1(\mathbf{n}) v_2(\mathbf{m})\rangle = -\mathbf{n}\cdot\mathbf{m}$. Now we can check this result against the CHSH inequality. We consider the case where $\mathbf{n}'$, $\mathbf{m}$, $\mathbf{n}$, $\mathbf{m}'$ are coplanar and separated by successive 45°; we obtain $|\langle B\rangle| = 2\sqrt{2}$ in clear violation of the CHSH inequality. Thus the combination of discrete outcomes and classical physics (in a statistical meaning) gives rise to a violation of the CHSH inequality. The following theorem holds: *Local realism, the transformation properties of vectors in classical physics (followed at least in the mean), and discrete outcomes cannot hold all together.* We also observe that the obtained result $\langle v_1(\mathbf{n}) v_2(\mathbf{m})\rangle = -\mathbf{n}\cdot\mathbf{m}$ coincides with predictions of ordinary quantum mechanics. Just starting from discrete outcomes and assuming transformation properties of vectors on average we have obtained the violations of the CHSH predicted by quantum theory. It is worth noting that the present result has been obtained without using Hilbert spaces , tensor products, Hermitian operators, Pauli matrixes. According to this analysis violations of Bell inequalities (usually regarded as the most striking departure of quantum mechanics from classical physics) is the only possibility for a physical system with zero total angular momentum and with dicotomic outcomes to follow the transformation properties of vectors. From this point of view local realistic theories are more nonclassical than quantum mechanics as they do not follow even in the mean the transformation rules of classical

physics.

The experimental tests of the CHSH inequalities confirm that nature choose to follow the laws of classical physics as far as allowed (in the mean) by the presence of discrete outcomes, giving up local realism [12, 13]. Here local realism seems to play the role of absolute time in special relativity. The theorem and this analogy suggest a formulation of quantum theory by means of only two general principles. A first principle accounts for many experimental evidences that microscopic systems if asked provide discrete outcomes. This may appear reasonable. For example, it seems to us reasonable that a light beam cannot be subdivided indefinitely. Feynman wrote: " It always bothers me that, according to the laws as we understand them today, it takes a computing machine an infinite number of logical operations to figure out what goes on in no matter how tiny a region of space, and no matter how tiny a region of time ... Why should it take an infinite amount of logic to figure out what one tiny piece of space/time is going to do?". The only possibility to describe what happens in a finite region of space with a finite amount of information is the assumption of discrete outcomes. Of course the concept of discretization does not produce automatically quantum physics; classical information theory, for example, even if based on discretization is not quantum. The theorem here presented indicates that continuous transformation properties of classical physical quantities under coordinate transformation may be the additional ingredient giving rise to quantum physics.

In 1999 Anton Zeilinger proposed the following information-based foundational principle for quantum mechanics [14]: "*The most elementary system carries just one bit of information*". As remarked by Zeilinger this principle is basic and elementary enough that it actually can serve as a foundational principle for quantum mechanics. Some of the essential features of quantum mechanics as the irreducible randomness of individual events, quantum complementary and quantum entanglement, arise in a natural way from it. Of course discrete outcomes would be a direct consequence of this principle. We adopt this principle as the first principle for quantum physics. As remarked above, the only possibility for a system with discrete outcomes to follow continuous classical transformations is to adhere to them in the mean. The second principle has to contain somewhat this adherence. We propose: *Physical systems satisfy the laws of classical physics in the mean when the first principle prevents a deterministic adherence.* This is a simple conservative principle preventing any conflict between the laws of classical physics (including relativity theories). After accepting

the two principles, the theorem becomes a direct consequence of them. Hence the two principles imply quantum entanglement and consequent violations of the CHSH inequality. In the following we will see as the principles also allow a direct derivations of other relevant and counterintuitive quantum results. We start considering what in ordinary quantum theory is the projection postulate. If we perform a first measurement on a physical system, we obtain one specific discrete outcome (as prescribed by the first principle). If we repeat again the same measurement (without affecting the system in the meanwhile between the two measurements), we will obtain deterministically the same discrete outcome obtained in the first measurement as happens in classical physics (according to the second principle). This is consequence of the fact that in this case the classical result doesn't conflict with discrete outcomes. As an example we may refer once again to the polarization of classical light. If we start with e.g. a V-polarized beam and send it to one or more PBS in the H/V basis, the polarization state will not be modified and the beam will always follow the V path.

We now begin from elementary systems and we face with the problem of describing classical properties, as the transformation of vectors under rotations, with only two possible outcomes $\pm a$ (1 bit). Specifically we assume that we obtained the outcome $+a$. Thus the system is oriented along the $\hat{\mathbf{n}}$ axis [19] with positive direction: $\mathbf{J} = \hat{\mathbf{n}}$. If we perform a polarization measurement along the $\hat{\mathbf{n}}$ axis, according to classical physics we obtain $\mathbf{J}\cdot\hat{\mathbf{n}} = a$. This result is in $\{\pm a\}$ hence, according to the second principle, it is also the quantum result. What happens if we choose a different detection axis $\hat{\mathbf{m}}$? According to classical physics we obtain [19] $\mathbf{J}\cdot\hat{\mathbf{m}} = a\hat{\mathbf{n}}\cdot\hat{\mathbf{m}} = a\cos\theta$, being θ the angle determined by the two axis. This result is not in $\{\pm a\}$ hence, according to the second principle, the quantum result follows the classical one only in the mean: $\langle \mathbf{J}\cdot\hat{\mathbf{m}}\rangle_{\hat{\mathbf{n}}+} = a\cos\theta$ and the expectation value can be expressed in terms of probabilities $P(\hat{\mathbf{n}}\pm, \hat{\mathbf{m}}\pm)$ as $\langle \mathbf{J}\cdot\hat{\mathbf{m}}\rangle_{\hat{\mathbf{n}}+} = a\langle \hat{\mathbf{n}}\cdot\hat{\mathbf{m}}\rangle \equiv aP(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}+) - aP(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}-)$. As a consequence we have the following equation

$$aP(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}+) - aP(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}-) = a\cos\theta\,. \tag{1}$$

By using that probabilities must sum to unity: $P(\hat{\mathbf{n}}\pm : \hat{\mathbf{m}}+) + P(\hat{\mathbf{n}}\pm : \hat{\mathbf{m}}-) = 1$, we can readily derive quantum probabilities

$$\begin{aligned} P(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}+) &= \cos^2(\theta/2) \\ P(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}-) &= \sin^2(\theta/2)\,. \end{aligned} \tag{2}$$

Analogously, starting from the outcome $-a$, we would obtain $P(\hat{\mathbf{n}}- : \hat{\mathbf{m}}\pm) = P(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}\mp)$. This result shows how the probabilistic nature of quantum theory descends directly from the two principles. In particular we derived the correct quantum probabilities prescribed by quantum mechanics starting from only two principles. We notice that, as expected, probabilities depend only on the relative angle between the two involved directions, hence exchanging the two directions does not modify probabilities. Also exchanging the initial and final outcomes does not modify probabilities: $P(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}-) = P(\hat{\mathbf{n}}- : \hat{\mathbf{m}}+)$. We interpret this finding as a sort of probabilistic reversibility, that is what survive of classical reversibility after the effects of the two principles. Once we obtain probabilities for a measurement of $\mathbf{J}$ along a given axis we are able to obtain information about its variance by deriving

$$\left\langle (\mathbf{J}\cdot\hat{\mathbf{m}})^2 \right\rangle_{\hat{\mathbf{n}}\beta} = \sum_{\alpha=\pm 1} (\alpha a)^2 P(\hat{\mathbf{n}}\beta : \hat{\mathbf{m}}\alpha)\,. \tag{3}$$

From probability conservation the above expression is a constant equal to a^2. As a consequence the expectation value of the square modulus of $\mathbf{J}$ results to be a scalar according to the second principle and in agreement with quantum mechanics: $\langle \mathbf{J}^2 \rangle = 3a^2$. The obtained results coincide with corresponding results obtained by using 2D Hilbert spaces and the theory of angular momentun of ordinary quantum theory. So as far as we calculate transition probabilities for vectors with two outcomes, it can be useful to use 2D Hilbert spaces just for all practical purposes. For example transition probabilities can be obtained in the usual mathematically elegant way as $P(\hat{\mathbf{m}}j : \hat{\mathbf{n}}k) = |\langle \hat{\mathbf{n}}k | \hat{\mathbf{m}}j \rangle|^2$, being $|\hat{\mathbf{m}}j\rangle$ a normalized vector in a 2D Hilbert space.

A direct consequence of the first principle is that elementary composite systems may be tought as constituted by two elementary systems and, hence, carry two bits. We consider measurements along a given $\hat{\mathbf{n}}$ axis on such systems. According to the first principle there are three different possibilities: $\pm 2a$ when both the values of the two elementary systems are equal and 0 when they assume opposite values. We start considering the outcome $+2a$ corresponding to the couple of states $\{\hat{\mathbf{n}}+\}_1 | \{\hat{\mathbf{n}}+\}_2$. What happens if we choose a different detection axis $\hat{\mathbf{m}}$? We derive the probabilities $P(\hat{\mathbf{n}}2 : \hat{\mathbf{m}}k)$ (with $k = \pm 2, 0$) of obtaining $\pm 2a, 0$ when performing measurements along $\hat{\mathbf{m}}$. The probability of obtaining $\pm 2a$ is given by the probability that both elementary systems give $\pm a$: $P(\hat{\mathbf{n}}2 : \hat{\mathbf{m}} \pm 2) = P_1(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}\pm) P_2(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}\pm) = (1 \pm \cos\theta)^2/4$. The outcome 0 arises if the first elementary outcome assumes values $\pm a$ and the second outcome gives opposite values $\mp a$. Hence,

$P(\hat{\mathbf{n}}2 : \hat{\mathbf{m}}0) = P_1(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}+)P_2(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}-) + P_1(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}-)P_2(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}+)$ and we obtain

$$P(\hat{\mathbf{n}}2 : \hat{\mathbf{m}}0) = \frac{1}{2}(1 - \cos^2\theta). \tag{4}$$

This result can also be obtained by exploting that probabilities must sum to one: $\sum_k P(\hat{\mathbf{n}}2 : \hat{\mathbf{m}}k) = 1$ (with $k = 0, \pm 2$). Analogous results can be derived for $P(\hat{\mathbf{n}} - 2 : \hat{\mathbf{m}}k)$. Oerived probabilities satisfy probabilistic reversibility: $P(\hat{\mathbf{n}} - 2 : \hat{\mathbf{m}}2) = P(\hat{\mathbf{n}}2 : \hat{\mathbf{m}} - 2)$. The consistency of this approach can be checked by using the derived probabilities to inspect the rotation properties of the expectation values $\langle \hat{\mathbf{n}} \pm 2 | \boldsymbol{J} | \hat{\mathbf{n}} \pm 2 \rangle$ and $\langle \hat{\mathbf{n}} \pm 2 | \mathbf{J}^2 | \hat{\mathbf{n}} \pm 2 \rangle$. They can be readily calculated by using the obtained probabilities and it turns out that the first is a vector and the second a scalar (equal to $8a^2$) consistently with the second principle and with QM. In order to complete we have to derive $P(\hat{\mathbf{n}}0 : \hat{\mathbf{m}}k)$. The physical state $\hat{\mathbf{n}}0$ brings some complication due to the fact that outcome zero can be obtained with two distinct possibilities: zero is obtained when the second system gives outcomes which are opposite to those of the first. In particular this outcome is produced by the two couples of elementary systems: $|n\pm\rangle_1 |n\mp\rangle_2$. Hence we do not know to which of the two couples a specific realization of the outcome zero corresponds. The most simple idea is to think to an equal mixture of the two systems, e.g. when once obtains zero, ones obtained or $|n+\rangle_1|n-\rangle_2$ either $|n-\rangle_1|n+\rangle_2$ each with a 50% of probability. Following this procedure we obtain $P(\hat{\mathbf{n}}0 : \hat{\mathbf{m}}2) = \left[P_1(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}+)P_2(\hat{\mathbf{n}}- : \hat{\mathbf{m}}+) + P_1(\hat{\mathbf{n}}- : \hat{\mathbf{m}}+)P_2(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}+)\right]/2$. Hence $P(\hat{\mathbf{n}}0 : \hat{\mathbf{m}}2) = P(\hat{\mathbf{n}}2 : \hat{\mathbf{m}}0)/2$ in contrast to previous cases. According to this result transitions $\pm 2a \to 0$ would be more favorite than transitions $0 \to \pm 2a$. This can be understood as a consequence of the hidden information on which of the two elementary systems gives $+a$. This derivation attributes elements of physical reality to the two elementary subsystems and opens the possibility of some hidden variables associated to the zero outcome. By using this proposed probability $P(\hat{\mathbf{n}}0 : \hat{\mathbf{m}}2) = \frac{1}{4}(1 - \cos^2\theta)$, we would obtain $\langle \hat{\mathbf{n}}0 | \mathbf{J}^2 | \hat{\mathbf{n}}0 \rangle = 4a^2 \neq \langle \hat{\mathbf{n}} \pm 2 | \mathbf{J}^2 | \hat{\mathbf{n}} \pm 2 \rangle$. This implies that a rotation can produce a change in the value of the scalar $\langle \mathbf{J^2} \rangle$ in contrast with the second principle. Again elements of reality, discrete outcomes, and rotation properties of vectors conflict. The correct transition probability can be derived regarding the components $\langle J_i^2 \rangle$ as the diagonal elements of a symmetric rank 2 tensor M_{ij}. We start from the state $\hat{\mathbf{z}}0$. We know that $M_{33} = 0$. We also know that $\mathrm{Tr}\mathbf{M} = \langle \mathbf{J^2} \rangle = 8a^2$. Symmetry implies $M_{11} = M_{22}$ and $M_{ij} = 0$ for $i \neq j$. According to the transformation rules for tensors, we obtain $M'_{33} = 4a^2 \sin^2\theta$ where θ is the angle between the $\hat{\mathbf{z}}$ and the new

measurement axis. According to our principles this result has to hold statistically, hence we have $P(\hat{\mathbf{n}}0 : \hat{\mathbf{m}}2) = P(\hat{\mathbf{n}}2 : \hat{\mathbf{m}}0) = \frac{1}{2}\sin^2\theta$. Thus also in this case reversibility holds. This result shows that we cannot attribute elements of physical reality to the two elementary subsystems. They cannot be interpreted as an equal mixture of the two systems but as an indistinguishable combination of them in such a way that the probability $P(\hat{\mathbf{n}}0 : \hat{\mathbf{m}}2)$ results as the sum of two contributions: $P_1(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}+)P_2(\hat{\mathbf{n}}- : \hat{\mathbf{m}}+) + P_1(\hat{\mathbf{n}}- : \hat{\mathbf{m}}+)P_2(\hat{\mathbf{n}}+ : \hat{\mathbf{m}}+)$. Indistinguishability gives rise to an additional factor two in the transition probability in analogy with constructive interference arising from coherent waves. It is remarkable that wave-like effects comes out by imposing only the rotation properties of vectors in presence of discrete outcomes. Indeed obtained results agree with the theory of angular momentun of ordinary quantum theory. Also in this case probabilities can be expressed as : $P(\hat{\mathbf{m}}j : \hat{\mathbf{n}}k) = |\langle \hat{\mathbf{n}}k|\hat{\mathbf{m}}j\rangle|^2$. Dealing with the Hilbert space, we notice that rotations mix the three states $|\hat{\mathbf{n}}, k\rangle$ spanning a 3D subspace of the 4D space. The remaining 1D subspace is spanned by the vector $|\psi^-\rangle = |\hat{\mathbf{m}}, 0\rangle = \frac{1}{\sqrt{2}}(|\hat{\mathbf{m}}+, \hat{\mathbf{m}}-\rangle - |\hat{\mathbf{m}}-, \hat{\mathbf{m}}+\rangle)$ that, of course, is orthogonal to the other three vectors. This state corresponds to the outcome zero. It is invariant if expressed in a different basis (as it is well known), thus produces only zero outcomes even if we change measurement axis. It gives $\langle\psi^-|\mathbf{J}\cdot\hat{\mathbf{m}}|\psi^-\rangle = 0$ and $\langle\psi^-|\mathbf{J}^2|\psi^-\rangle = 0$, hence it provides the realization of the null vector within two elementary systems. Quantum probabilities for higher order angular momenta as well as for more complex operators as symmetric tensors can be obtained following this approach.

In conclusion, our reformulation of QM based just on bits and classical physics, provides a physical explanation to the most puzzling quantum features and specifically to entanglement. We hope that this conceptual understanding will be useful for the developing field of quantum information. Of course we stress that these results are not a complete reformulation and constitute only a promising starting point. Despite we were able to reproduce results of the angular momentum in ordinary quantum theory, it should be evident that the present approach is far to be innocuous: Within this approach there is no room to quantum observables displaying continuous outcomes in contrast to ordinary QM. This would imply that a tiny region of space can support only a finite amount of information. This consideration, descending naturally from the present approach, constitutes the common key ingredient of theories trying to match quantum theory with general relativity. We belive that the approach presented here may provide a rigorous route towards the solution of this

problem.

We thank O. Maragò and S. Portolan for helpful discussions and comments.

[1] Clauser, J. F. & Shimony, A. *Bell's theorem: experimental tests and implications.* Rep. Prog. Phys. **41**, 1883-1927 (1978).

[2] Bennett, C. & DiVicenzo, D. *Quantum information and computation.* Nature **404**, 247255 (2000).

[3] Nielsen, M. A. & Chuang, I. L. *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).

[4] Bell, J. S. *Speakable and unspeakable in quantum mechanics*, 2nd edition (Cambridge University Press, Cambridge, 2004).

[5] Amelino-Camelia, G. *Quantum theorys last challenge.* Nature **408**, 661-664 (2000).

[6] Dreyer, O. *Quasinormal Modes, the Area Spectrum, and Black Hole Entropy.* Phys. Rev. Lett. **90**, 081301-1 081301-4 (2003).

[7] Baez, J. *The quantum of area?* Nature **421**, 702-703 (2003).

[8] Rovelli, C. *Quantum gravity.* (Cambridge University Press, Cambridge, 2004).

[9] Schlosshauer, M. *Decoherence, the measurement problem, and interpretations of quantum mechanics.* Rev. Mod. Phys **76**, 1267 (2004).

[10] Einstein, A., Podolsky, B. & Rosen, N. *Can quantum-mechanical description of physical reality be considered complete?* Phys. Rev. **47**, 777 (1935).

[11] Bell, J. S. *On the Einstein-Podolsky-Rosen paradox* Physics **1**, 195-200 (1964).

[12] Aspect, A. *Bell's inequality test: more ideal than ever* Nature **398**, 189-190 (1999), and references herein cited.

[13] Rowe, M. A. *et al. Experimental violation of a Bell's inequality with efficient detection*, Nature **409**, 791-794 (2001).

[14] Zeilinger, A. *A foundational principle for quantum mechanics.* Foundations of Physics **29** 631 (1999).

[15] C. A. Fuchs, C. A. *Quantum foundations in the light of quantum information*, in *Decoherence and its implications in quantum computation and information transfer*, 38-82. Proceedings of the NATO Advanced Research Workshop, Mykonos Greece, June 2530, 2000, edited by A.

Gonis and P. E. A. Turchi (IOS Press, Amsterdam, 2001), also quant-ph/0106166.

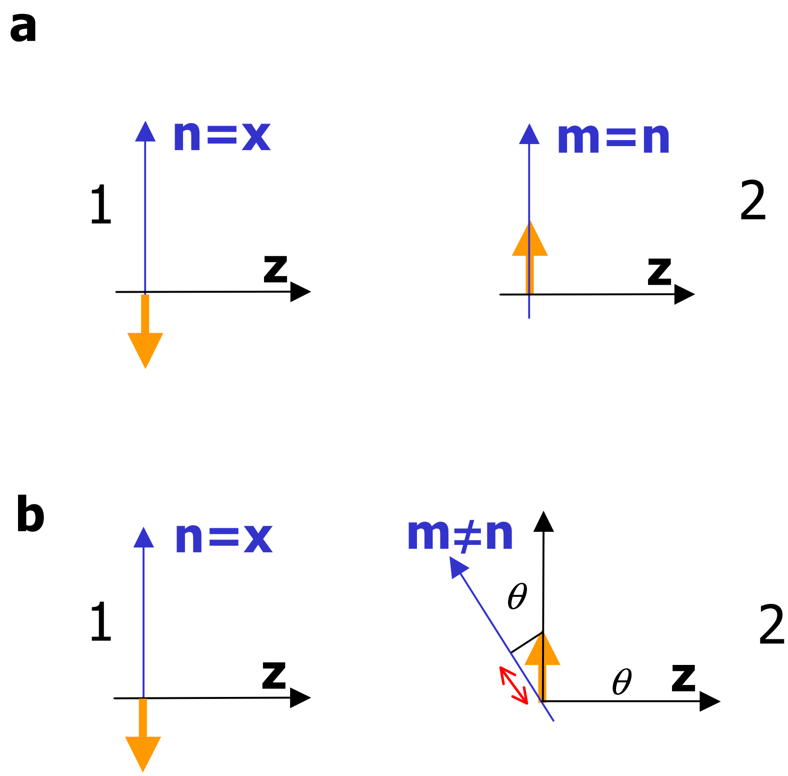
[16] Brukner, Č. & Zeilinger, A. *Information and fundamental elements of the structure of quantum theory* in *Time, quantum and information* . (Springer Verlag 2004), also e-print quant-ph/0212084, and references herein cited.

[17] Landau, L. D. & Lifshitz E. M. *Quantum mechanics.*, 3rd edition, Pergamon Press (1977).

[18] Clauser, J. F. Horne, M. A. Shimony, A., & Holt, R. A. *Proposed experiment to test local hidden-variables theories.* Phys. Rev. Lett. **49**, 1804-1807 (1969).

[19] We exploit the fact that $v(\hat{\mathbf{n}}) \equiv \mathbf{J} \cdot \hat{\mathbf{n}} = \pm 1$ implies that $\langle \mathbf{J} \cdot \hat{\mathbf{n}}_\perp \rangle = 0$ (being $\hat{\mathbf{n}}_\perp$ orthogonal to $\hat{\mathbf{n}}$). If not, a rotation could give rise to expectation values $\langle v(\hat{\mathbf{n}}) \rangle$ outside the interval between the possible outcomes.

Fig. 1 Scheme describing measurements for two particles with dicotomic outcomes and with null total angular momentum: (a) view of one of the two possible results when the two observers choose the same axis. (b) What happens when observer 2 choose a different rotation axis, according to the transformation properties of vectors under rotation.


a
n=x
1
z
m=n
2
z
b
n=x
1
z
m≠n
θ
2
θ
z